Operation of Coupled Multi-Owner District Heating Networks via Distributed Optimization

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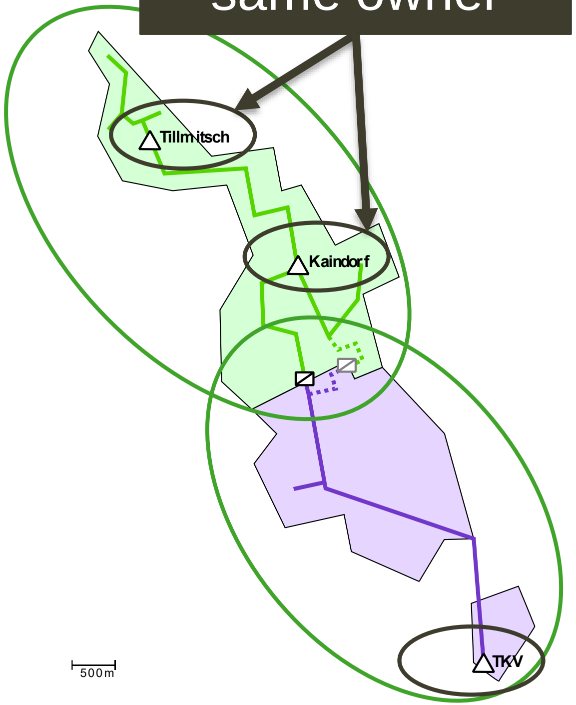
Valentin Kaisermayer, Daniel Muschick, Martin Horn and Markus Gölles

Motivation
Coupling of District Heating Networks

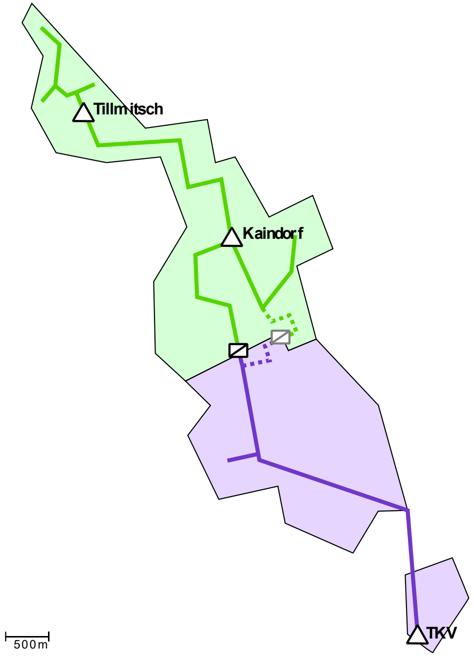
- Three district heating networks (near Leibnitz in Styria, Austria) have grown and reached the boundaries of their neighbouring networks
- Two owners operate the networks
- Waste heat in the south should be used to supply the northern networks in summer

Operated by the same owner



Motivation cont.
Coupling of District Heating Networks

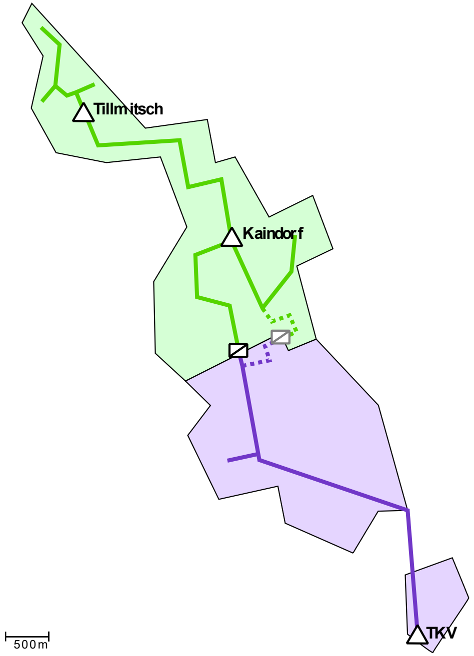
- **Each network (agent) has local**
  - Production, demand & storage
  - Different technologies and fuel prices
- **Networks are coupled via heat exchangers**
  - Incur losses
  - Operating strategy influences both connected networks
- **Networks are bound by existing contracts**



Objective

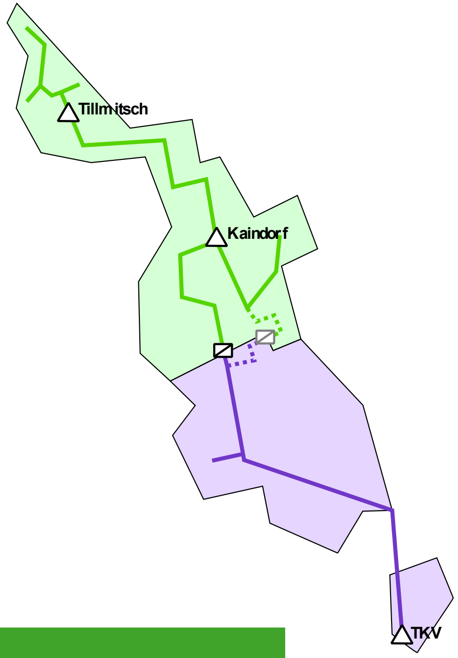
- **How should the individual network operator run his/her network?**
  - Heat exchanger is like an additional technology for unit commitment & dispatch
  - BUT not only own restrictions matter, but also those of the heat provider

Find co-ordinated operating strategies
that maximize the profit
for every individual network



Problems during Operation

- **Costs of heat are variable**
  - Depending on current production / demand / storage: transmitting more heat would require running the backup boiler, which is more expensive
- **Direction of heat transport cannot change too often**
  - long transmission lines would cool down
- **Hydraulic limitations do not allow supply of all consumers**
  - Pipes towards heat exchanger might not have sufficient capacity



Typical Solution: Solve Optimization Problem

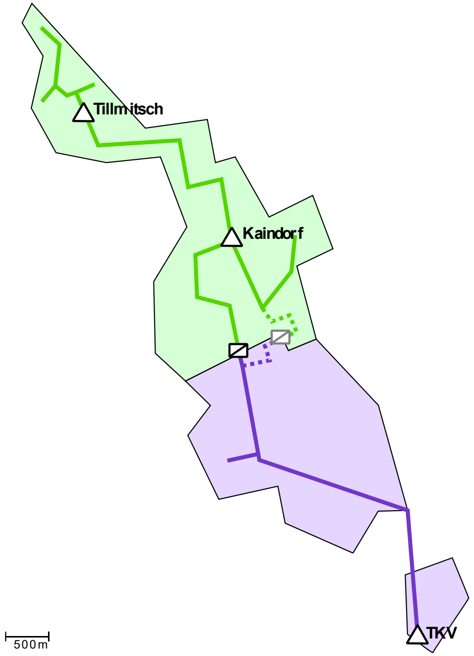
Mathematical Representation
Local Problems

- Optimization problem for each network:
- Local cost function & constraints

$$\min_{\boldsymbol{x}_i \in \mathcal{X}_i} f_i(\boldsymbol{x}_i)$$

- Coupling via global constraints

$$\sum_{i=1}^{N} \boldsymbol{A}_i \boldsymbol{x}_i = \boldsymbol{b}$$



How to formulate the global optimization problem?

Mathematical Representation
Two coupling strategies

Cooperative coupling

- Agents minimize a **global** objective

Non-cooperative coupling

- Agents minimize only their **local** objective

Mixture of both (some cooperate, some do not)?

BEST Bioenergy and Sustainable Technologies

Mathematical Representation
Two coupling strategies cont.

Cooperative coupling
- Each grid has local constraints and local objective function
- Agents **minimize global cost function**

$$\min_{\boldsymbol{x}_i \in \mathcal{X}_i} \quad \sum_{i=1}^{N} f_i(\boldsymbol{x}_i)$$
$$\text{s.t.} \quad \sum_{i=1}^{N} \boldsymbol{A}_i \boldsymbol{x}_i = \boldsymbol{b}$$

Global optimum

Non-cooperative coupling
- Each grid has local constraints and local objective function
- Each agent **minimizes only local cost function**

$$\min_{\boldsymbol{x}_i \in \mathcal{X}_i} \quad f_i(\boldsymbol{x}_i), \quad i = 1, \ldots, N$$
$$\text{s.t.} \quad \sum_{i=1}^{N} \boldsymbol{A}_i \boldsymbol{x}_i = \boldsymbol{b}$$

Nash equlibrium

Mathematical Representation
Excursion: Nash Equilibrium

Nash Equilibrium

- N-player game

$$\min_{\boldsymbol{x}_i \in \mathcal{X}_i} f_i(\boldsymbol{x}_i, \boldsymbol{x}_{-i}), \quad i = 1, \ldots, N$$

$$f_i(\boldsymbol{x}_i^*, \boldsymbol{x}_{-i}^*) \leq f_i(\boldsymbol{x}_i, \boldsymbol{x}_{-i}^*), \quad \forall \boldsymbol{x}_i \in \mathcal{X}_i$$

Generalized Nash Equilibrium

- Constrained N-player game

$$\min_{\boldsymbol{x}_i \in \mathcal{X}_i(\boldsymbol{x}_{-i})} f_i(\boldsymbol{x}_i, \boldsymbol{x}_{-i}), \quad i = 1, \ldots, N$$

What agent *i* can do depends on what the others do

$$f_i(\boldsymbol{x}_i^*, \boldsymbol{x}_{-i}^*) \leq f_i(\boldsymbol{x}_i, \boldsymbol{x}_{-i}^*), \quad \forall \boldsymbol{x}_i \in \mathcal{X}_i(\boldsymbol{x}_{-i})$$

“No one can improve his/her objective by **unilaterally** changing his/her strategy”

Cooperative Coupling

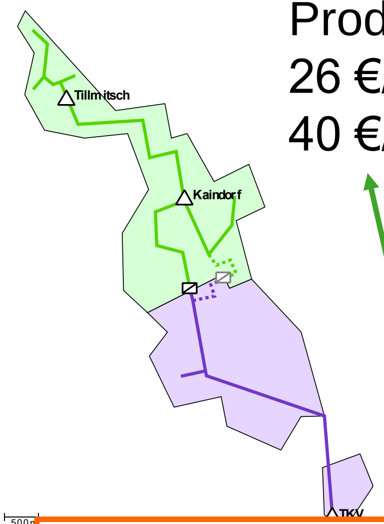
$$\min_{\boldsymbol{x}_i \in \mathcal{X}_i} \quad \sum_{i=1}^{N} f_i(\boldsymbol{x}_i)$$

$$\text{subject to} \quad \sum_{i=1}^{N} \boldsymbol{A}_i \boldsymbol{x}_i = \boldsymbol{b}$$

- Separable programme
- Solution is **global optimum**

Large (separable) opt. problem

- **When is this not the „best“ solution?**



Global optimum: Produce everything in the south!

Non-Cooperative Coupling

$$\min_{\boldsymbol{x}_i \in \mathcal{X}_i} \quad f_i(\boldsymbol{x}_i), \quad i = 1, \ldots, N$$

$$\text{subject to} \quad \sum_{i=1}^{N} \boldsymbol{A}_i \boldsymbol{x}_i = \boldsymbol{b}$$

How to simultaneously solve this?

- N-player game
- Solution is Nash equilibrium

N coupled opt. problems

x Iterative algorithm
x No unique solution!
x No notion of "better" or "worse" solution

- Augmented Lagrange functions

$$\mathcal{L}_i(\boldsymbol{x}_i, \boldsymbol{\lambda}_i, \boldsymbol{x}_{-i}) = f_i(\boldsymbol{x}_i) + \boldsymbol{\lambda}_i^T \left(\boldsymbol{A}_i \boldsymbol{x}_i + \sum_{j \neq i} \boldsymbol{A}_j \boldsymbol{x}_j - \boldsymbol{b} \right) + \frac{\rho}{2} \left\| \boldsymbol{A}_i \boldsymbol{x}_i + \sum_{j \neq i} \boldsymbol{A}_j \boldsymbol{x}_j - \boldsymbol{b} \right\|_2^2$$

- Dual ascent

$$\boldsymbol{x}_i^{(k+1)} = \underset{\boldsymbol{x}_i \in \mathcal{X}_i}{\operatorname{argmin}} \ \mathcal{L}_i(\boldsymbol{x}_i, \boldsymbol{\lambda}_i^{(k)}, \boldsymbol{x}_{-i}^{(k)})$$

$$\boldsymbol{\lambda}_i^{(k+1)} = \boldsymbol{\lambda}_i^{(k)} + \rho \left(\boldsymbol{A}_i \boldsymbol{x}_i^{(k+1)} + \sum_{j \neq i} \boldsymbol{A}_j \boldsymbol{x}_j^{(k)} - \boldsymbol{b} \right)$$

Example
Two-player Game

- Player 1

$$f_1(x_1) = \min_{x_1 \geq 1} \quad x_1 x_2$$
$$\text{subject to} \quad x_1 + x_2 \leq 10$$

I will make x_1 as small as possible!

- Player 2

$$f_2(x_2) = \min_{x_2 \geq 1} \quad -x_1 x_2$$
$$\text{subject to} \quad x_1 + x_2 \leq 10$$

I will make x_2 as big as possible!

Example
Two-player Game

- **Player 1**

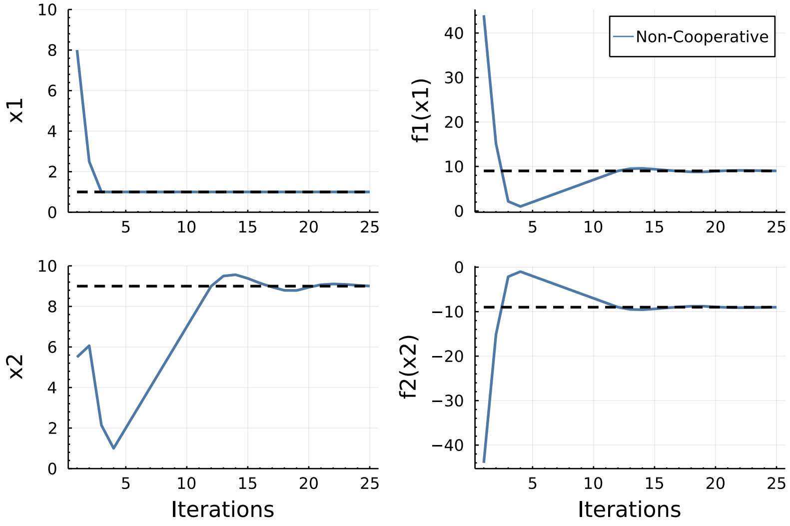
$$f_1(x_1) = \min_{x_1 \geq 1} \quad x_1 x_2$$
$$\text{subject to} \quad x_1 + x_2 \leq 10$$

- **Player 2**

$$f_2(x_2) = \min_{x_2 \geq 1} \quad -x_1 x_2$$
$$\text{subject to} \quad x_1 + x_2 \leq 10$$

Cooperative problem not well defined

$$f_1(x_1) + f_2(x_2) \equiv 0$$

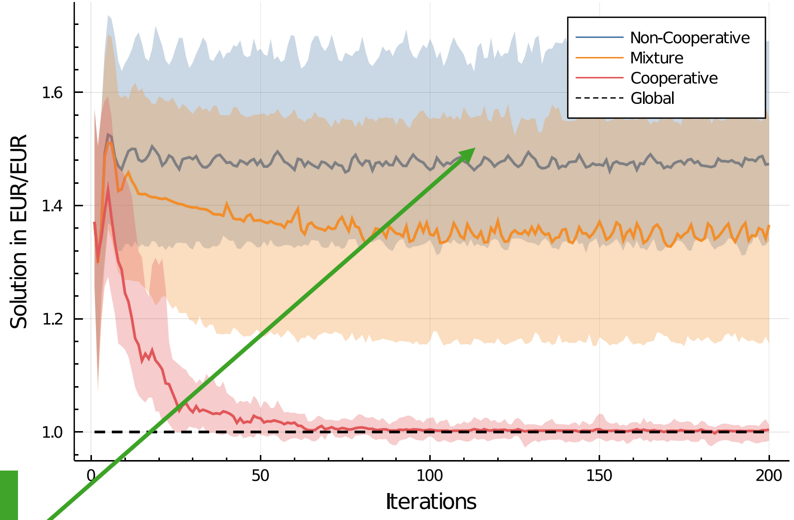


Simulation Study

Three grids & two owners

Test Problems

- All three grids **cooperative**
- All three grids **non-cooperative**
- The two grids that belong to the same owner cooperate; the third does not (**mixture**)



True Nash equilibrium not known (in general hard to compute)

Conclusions & Outlook

- Operating coupled networks cannot easily be described as an optimization problem
  - Competing interests are cancelled out in the cost function
- An iterative algorithm can be used to solve for **Nash equilibria** instead
  - No guarantee that these represent „good“ operating strategies
  - Further investigation into the role of the initial solution
  - Very similar algorithm to distributed optimization
- Designing **contracts** between operators is tricky
  - Which price if there are multiple possible sources?
  - Possibly the operating strategy must already be part of the contract?

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Daniel Muschick
daniel.muschick@best-research.eu
Tel.: + 43 5 02378-9248

Markus Gölles
markus.goelles@best-research.eu
Tel.: + 43 5 02378-9208